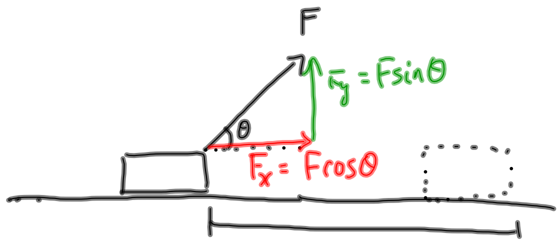
How do we calculate work when the force & displacement are not in the same direction?

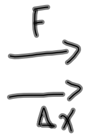


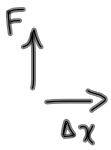
$\Delta W = F \Delta x$ ← F must be in the same direction as Δx

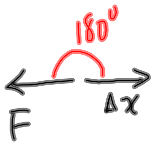
$\Delta W = (F\cos\theta)\Delta x$

$\Delta W = F\Delta x\cos\theta$

$\Delta W = Fs\cos\theta$ ← Data Booklet.

If  $\theta = 0°$ and $\Delta W = Fs$ (maximum work)

If  $\theta = 90°$ and $\Delta W = 0$ (no work)

If  $\theta = 180°$ and $\Delta W = -Fs$ (negative work)

Looking back at:

Calculate the work done by a net force of 10N which slows a moving object from 50 ms^{-1} to rest over a distance of 25m.

$\Delta W = Fs\cos\theta$

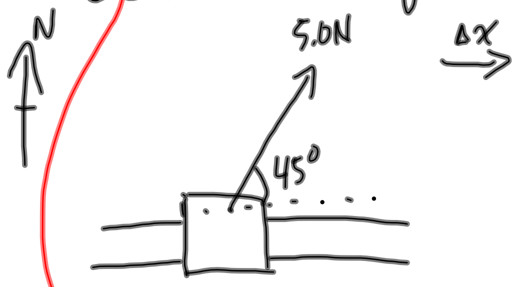
$\Delta W = (10N)(25m)\cos 180°$

$\Delta W = -250J$

The kinetic energy decreased by 250J. This 250J was transferred to internal energy.

Calculate the work done and describe the energy changes in each of the following:

① A force of 5.0N northeast moves an object a distance of 3.0m at a constant speed along an east-west track



$\Delta W = Fs\cos\theta$

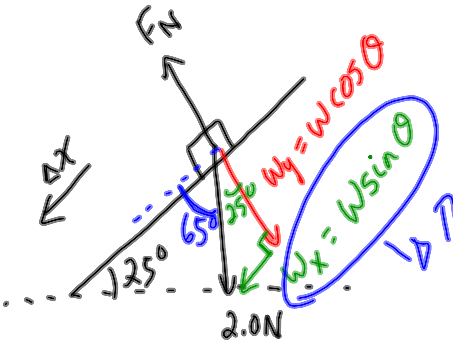
$\Delta W = (5.0N)(3.0m)\cos 45°$

$\Delta W = 11J$

→ no change in kinetic energy

→ internal energy

② A body whose weight is 2.0N slides 10m down a frictionless slope inclined at 25° to the horizontal.



→ This component of the weight does the work.

$\Delta W = F s$ ← the component of the force parallel to incline.

$\Delta W = (2.0N)(\sin 25°)(10m)$

$\Delta W = 8.5J$

gravitational potential → kinetic

(loses 8.5J) (gains 8.5J)

OR: $\Delta W = Fs\cos\theta$ ← between the force and the direction of the displacement

$\Delta W = (2.0N)(10m)\cos 65°$

$\Delta W = 8.5J$

③ A net force of 2.0×10^3 N acts for 5.0 s on a body of mass 2.0×10^2 kg which is initially at rest.

$\vec{F}$ $\vec{\Delta x}$

$\Delta W = Fs$?? we need to find s !

$F_{net} = ma$

$a = \frac{F_{net}}{m}$

$a = \frac{2.0\times10^3 N}{2.0\times10^2 kg}$

$\boxed{a = 10 ms^{-2}}$

$s = \cancel{ut}^{\,0} + \frac{1}{2}at^2$

$s = \frac{1}{2}(10ms^{-2})(5.0s)^2$

$\boxed{s = 125m}$

work → kinetic energy

$\Delta W = Fs$

$\Delta W = (2.0\times10^3 N)(125m)$

$\boxed{\Delta W = 2.5\times10^5 J}$

④ A girl pushes an object 5.0m along a horizontal road with a force of 100N against a friction force of 80N.

girl: $\vec{F}$ $\vec{\Delta x}$

$\Delta W = Fs$

$\Delta W = (100N)(5.0m)$

$\boxed{\Delta W = 500J}$

girl's chemical energy ↓ work ↓ kinetic energy.

friction: $\overleftarrow{F}$ $\vec{\Delta x}$

$\Delta W = Fs\cos\theta$

$\Delta W = (80N)(5.0m)\cos 180^\circ$

$\boxed{\Delta W = -400J}$

friction ↓ work ↓ internal energy

500J (girl) → 400J (internal)

500J (girl) → 100J (kinetic)

↳ work done by the net force

$\Delta W = F_{net}\, s$

$\Delta W = (20N)(5.0m)$

$\boxed{\Delta W = 100J}$

The object gains 100J of kinetic energy ←

(If there were no friction, the object would have gained 500J of kinetic energy)

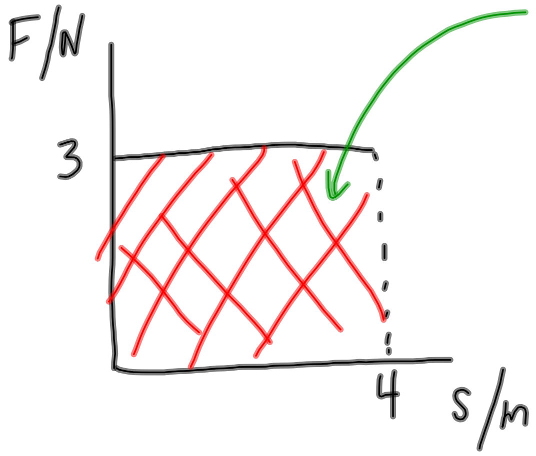
⑤ A man holds a load of 1000N stationary above his head.

$\Delta W = 0J$

man's chemical potential energy → internal

Force-Distance Graph

Constant Force:



Area of rectangle = $l \times w$

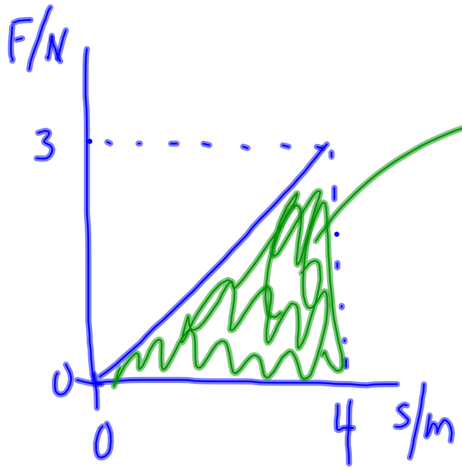
Are = $F s$ ← work!

The area under a Force distance graph is Work!

$\Delta W = (3N)(4m)$

$\Delta W = 12J$

Consider the graph for a force that is constantly changing:

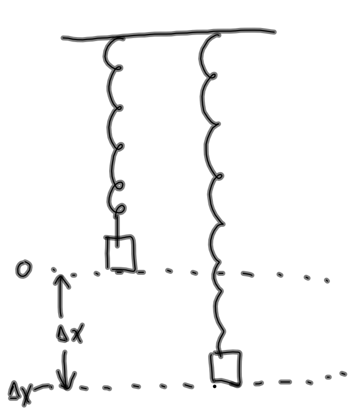


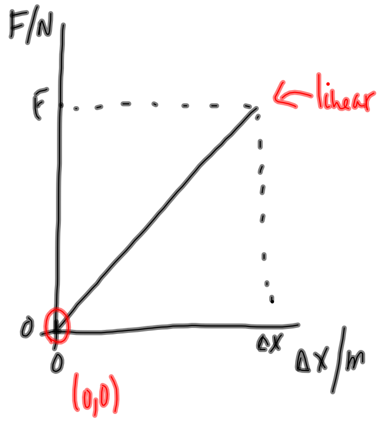
Area = ΔW

$\Delta W = \frac{1}{2}(3N)(4m)$

$\Delta W = 6J$

Hooke's Law: Extension of a Spring





$$F \propto \Delta x$$

Hooke's Law → $$\boxed{F = k\Delta x}$$

k is called the <u>force constant</u> (units: $N\,m^{-1}$)

k is the slope on the F-Δx graph

ΔW is the area under the graph

ΔW = Area

$\Delta W = \frac{1}{2} F \Delta x$ but $F = k\Delta x$

(this is the force to hold the spring at a distance of Δx from eq)

$\Delta W = \frac{1}{2}(k\Delta x)(\Delta x)$

$\Delta W = \frac{1}{2}k(\Delta x)^2$

This work is done to give the spring elastic potential energy

$$\boxed{E_p = \frac{1}{2}k(\Delta x)^2}$$

** <u>NOT</u> in the Data Booklet

** you must be able to derive or memorize!

Example

The force constant of a spring in a car suspension is $5.0 \times 10^4 \, Nm^{-1}$
How much work is done by a force in compressing the spring by 4.0 cm? How much energy is stored in the spring?

0.040 m

$$E_p = \frac{1}{2}k(\Delta x)^2$$

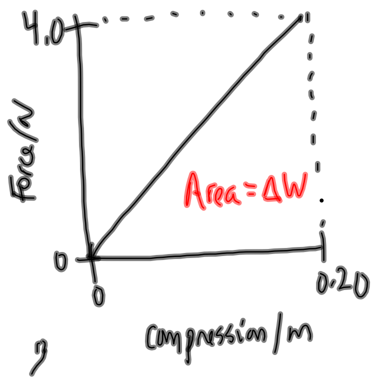
$$E_p = \frac{1}{2}(5.0 \times 10^4 \, Nm^{-1})(0.040m)^2$$

$$E_p = 40J$$

If there is 40J stored in the spring as elastic potential energy, then 40J of work was done in compressing the spring.

Example

Graph shows how the compression of a spring depends on the force applied to it.



- How much work is done in compressing the spring by 0.20m?

$$\Delta W = Area$$

$$\Delta W = \frac{1}{2}(4.0N)(0.20m)$$

$$\Delta W = 0.40J$$

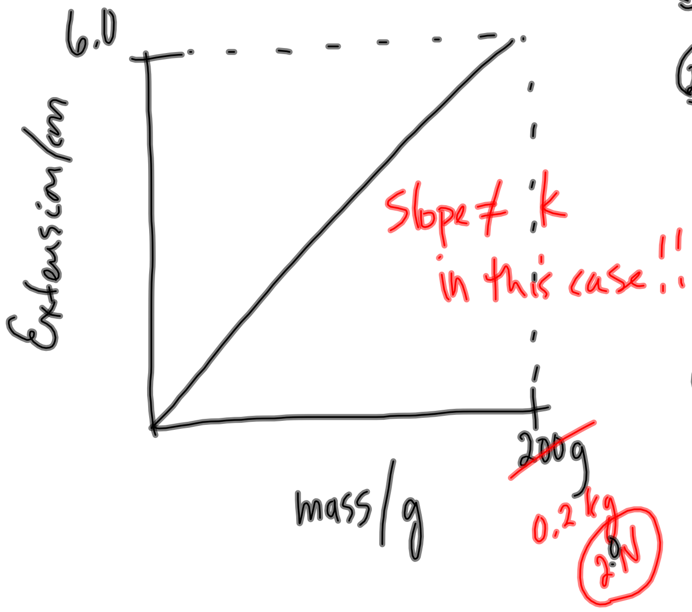
you did 0.40J to compress the spring.

What is the potential energy stored in the compressed spring?

0.40J

Example

different masses → stretch of spring.



Calculate:

① the force constant

② the work done to stretch 6.0 cm

(take $g = 10\,\text{ms}^{-2}$)

①
$$F = k\Delta x$$
$$k = \frac{F}{\Delta x}$$
$$k = \frac{2.0\,\text{N}}{0.060\,\text{m}}$$
$$k = 33\,\text{N}\,\text{m}^{-1}$$

②
$$\Delta W = \frac{1}{2}k(\Delta x)^2$$
$$\Delta W = \frac{1}{2}(33\,\text{N}\,\text{m}^{-1})(0.060\,\text{m})^2$$
$$\Delta W = 0.060\,\text{J}$$